Strömungen in porösen Medien

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PART I

- What are Porous Media ?
  - Characterization of Porous Media
  - Fields of Interest / Applications
  - Length scales / Hierarchy of Heterogeneity
  - Properties of Porous Media
  - Some Historical Aspects
- Momentum Transport in Porous Media: single phase flows
  - Empirical Laws: Darcy Law
  - Permeability
- Energy Transport in Porous Media
  - Conductive Transport
  - Radiative Transport
- Combustion in porous media

Characterization of Porous Media

- Heterogeneous system of solid matrix and void space with fluid involving two or three phases:

  solid - liquid
  solid - gaseous
  solid - gaseous - liquid

- Consolidated:
  fixed structures
- Static structures
- Regular shaped
- Deterministic

- Non-consolidated:
  loosely packed beds
- Non-static structures
- Highly irregular
- Statistical

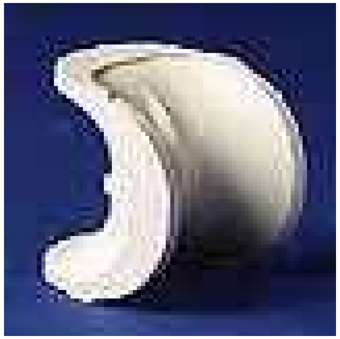
Forms of Porous Media

- naturally formed
  - rocks, sand beds, sediments, ground water transport
  - sponge-like structures, lung
  - woods, plants

- fabricated, artificial
  - catalytic pellets, Raschig rings, fixed beds of particles
  - wicks, filaments
  - insulation
  - foam-like structures
  - regular structures, e.g. monoliths
  - materials: metals, ceramics, glass,

Packing material

Raschig-Ringe



Berl-Sattel

Pall-Ringe

Top-Pak

VSP

Hacketten

Fields of Interest / Applications

- Chemical Engineering
  - catalytic and inert packed bed reactors (gaseous, aqueous)
  - filtering, drying, trickle bed reactors
  - catalytic converters
  - adsorption / desorption at surfaces
- Environmental / Geological
  - groundwater transport, contamination migration
  - salt water intrusion
  - waste disposal
- Petroleum
  - Oil / Gas flows in reservoirs

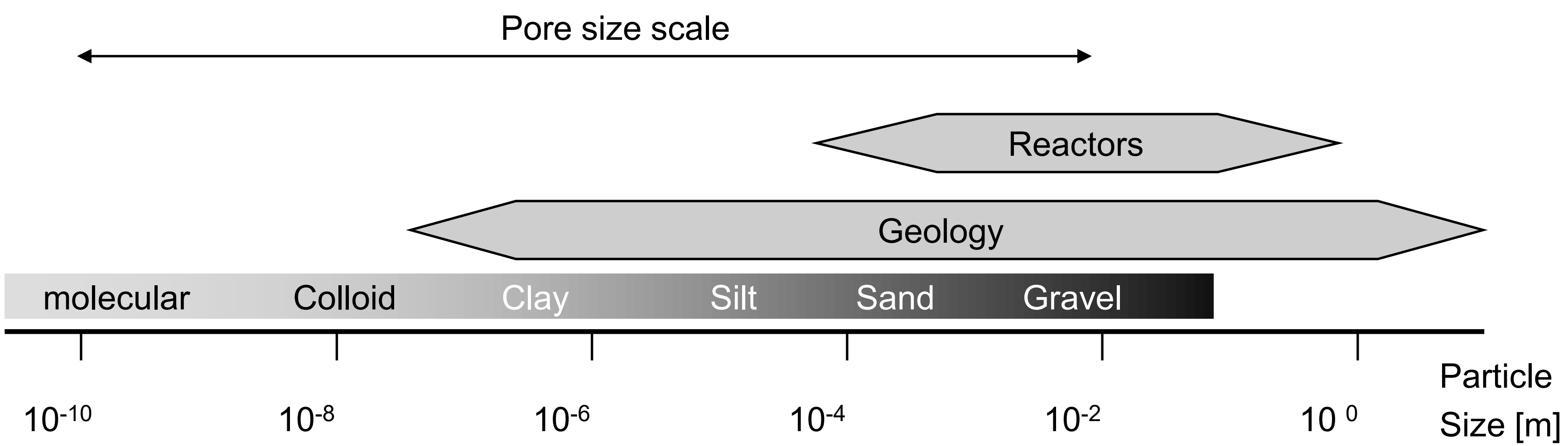
Fields of Interest / Applications

- Mechanical Engineering
  - Single- and two phase transpiration cooling
  - Insulation, thermal protection systems
  - Radiant porous burners
  - Enhancing heat transfer by surface modification
  - Preheating, flame stabilization
  - Absorbing and storing solar energy
  - Catalytic converters and soot traps

- Medicine / Biology
  - Exchange processes in human lounge
  - Transport in fibers / plants

Length scales / Hierarchy of Heterogeneity

- microscopic, ultra-micropore scale, pore scale :
  1-10 Å - Millimeter
- macroscopic scale: system dimension
- megascopic scale: dimension of entire reservoirs
- gigascopic scale: landscapes, rivers



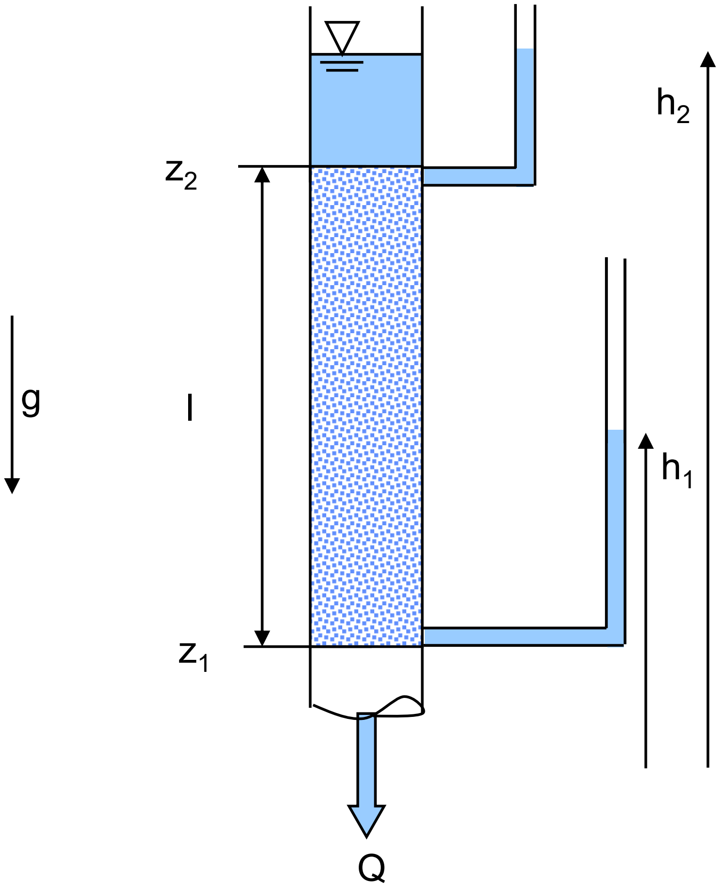
Characterization of Porous Media

$$\varepsilon = \text{porosity} = \frac{\text{fraction of (connected) volume occupied by fluid}}{\text{total volume of solid matrix and fluid}}$$

Foam metal	0.98
Fiberglas	0.88 - 0.93
Silica grains	0.65
Raschig rings	0.56 - 0.65
Crushed rock	0.44 - 0.45
Sand	0.37 - 0.50
Spherical packing	0.37 - 0.47
Cigarette filters	0.17 - 0.49
Coal	0.02 - 0.12
Concrete	0.02 - 0.07

Darcy's law: 1856

- experiment to determine the **bulk resistance** of a fluid through porous media
- assuming isotropic media
- non-consolidated, isotropic, small particles
- gravity driven flow
- internal surface much larger than confining domain
  => dominance of shear forces
- measurement of flow rates and pressure difference
- low pore velocities



Darcy's law: from measurements

$$Q = -A\,K\,\frac{\rho}{\mu}\,\frac{h_2 - h_1}{z_2 - z_1} \qquad U_D = -K\,\frac{\rho}{\mu}\,\frac{dh}{dz}$$

$$p = \rho g h - \rho g z \qquad \frac{dp}{dz} = \rho g\,\frac{dh}{dz} - \rho g$$

Q *volume flux*

A *area*

U_D *spec. volume flux per area*

g *gravity constant*

p *pressure*

k capilarity $= \dfrac{K}{g}$

$$U_D = \frac{k}{\mu}\left(\rho\vec{\mathbf{g}} - \frac{dp}{dz}\right)$$

$$U_D = -\frac{k}{\mu}\,\frac{dp}{dz}, \quad \text{if no gravity}$$

Darcy’s law: for multidimensional flows

in general coordinates for isotropic media

$$\vec{\mathbf{U}}_D = -\frac{k}{\mu}\nabla p$$

for an-isotropic media

$$\vec{\mathbf{U}}_D = -\frac{\mathbf{K}}{\mu}\nabla p \qquad \mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

for orthotropic media

$$U_{D,i} = -\frac{k_i}{\mu}\frac{dp}{dx_i} \qquad k_{ii} = k_i$$

Permeability: [m^2]

some examples from measurements

Cigarette	$1.1 \cdot 10^{-9}$
Sand	$2,0 \cdot 10^{-11}$ - $2,0 \cdot 10^{-10}$
Fiberglas	$2,4 \cdot 10^{-11}$ - $5,1 \cdot 10^{-10}$
Sandstone	$5,0 \cdot 10^{-16}$ - $3,0 \cdot 10^{-12}$
Dolomite	$\sim 10^{-15}$
Rock Salt	$\sim 10^{-17}$
	$5,0 \cdot 10^{-16}$ - $3,0 \cdot 10^{-12}$
	$5,0 \cdot 10^{-16}$ - $3,0 \cdot 10^{-12}$
	$5,0 \cdot 10^{-16}$ - $3,0 \cdot 10^{-12}$

1 Darcy $\sim 10^{-12}$ m^2

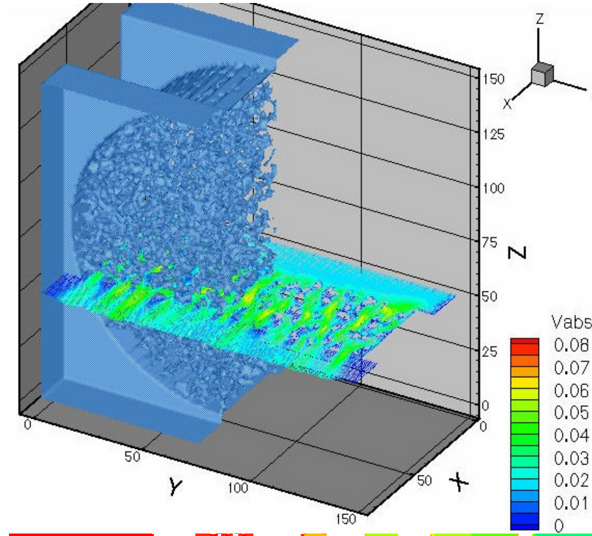
Permeability

- is a measure for the bulk conductance of the flow
- describes the bulk hydrodynamics (if Darcy law is valid)

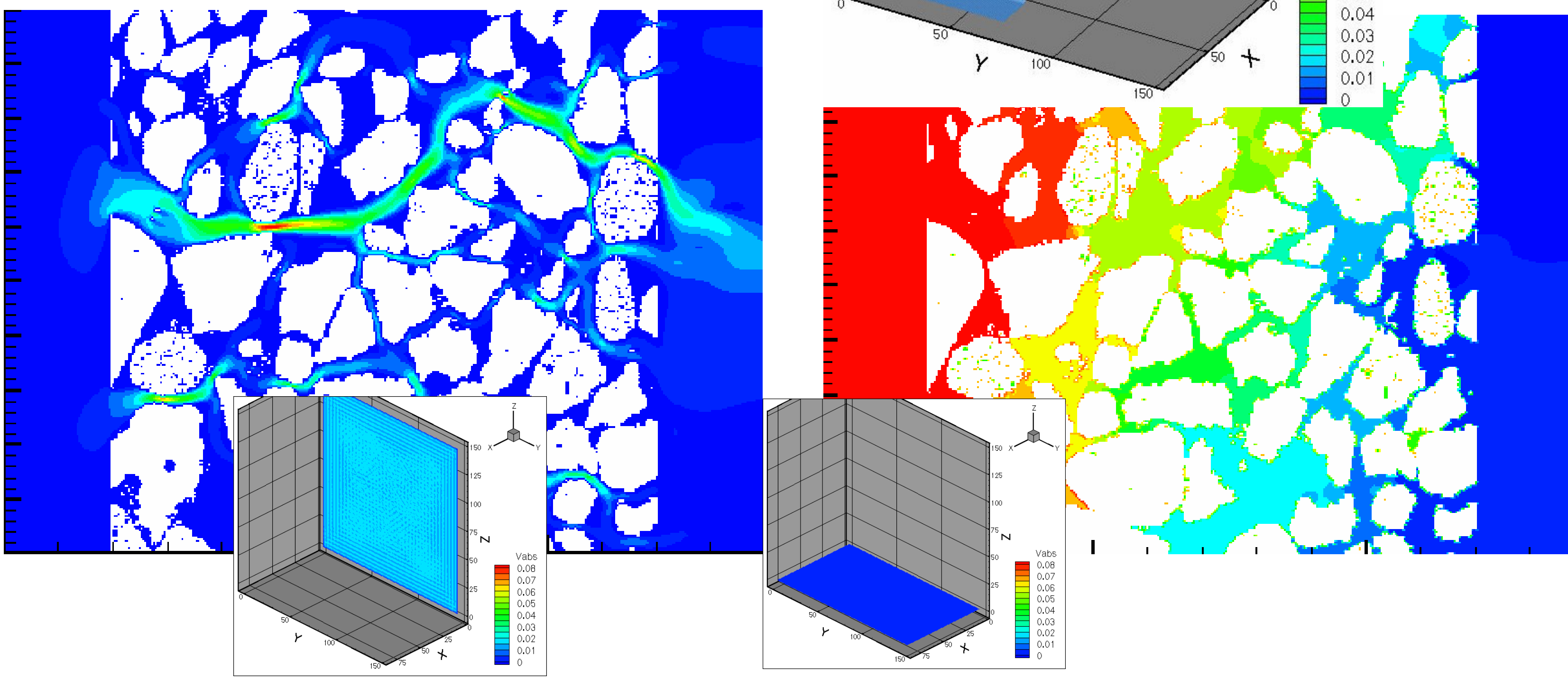
Estimation of Permeability

- from measurements of bulk properties
  - heuristic method
- from models
  - capillary models
  - hydraulic radius models
  - drag models
- from first principles
  - solving mass and momentum conservation at pore level
  - averaging the conservation equations

- Channeling effect in real probes of porous media (from numerical solution of Navier-Stokes Equations)



velocity magnitude



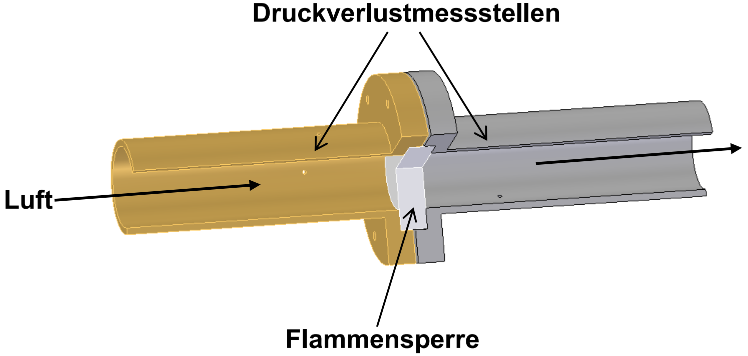
Flows with higher flow rates

- in turbulent flows, friction mainly depends on wall roughness

- pressure loss of a porous structure can be described by the Darcy equation an the Forcheimer extension

$$\frac{dp}{dz} = \underbrace{-\frac{\mu}{k_{lam,i}} U_D}_{\text{Darcy}} \underbrace{- \frac{\rho}{k_{turb,i}} U_D^2}_{\text{Forchheimer}}$$

- determination of pressure loss of real structures can be derived experimentally or numerically



<u>Flow Regimes</u> (in terms of pore level Reynoldsnumber)

- Re < 1: Darcy or creeping flow regime
  - viscous forces dominate
  - flow dominated by particle geometry on pore level
- 1-10 < Re < 150: Inertial flow regime
  - staedy nonlinear laminar flow
  - formation of boundary layer
- 150 < Re < 300: Unsteady laminar flow regime
  - wake instabilities, oscillations (~1Hz)
- 300 < Re: Unsteady chaotic (turbulent) regime
  - asymptotic behaviour (see Ergun equation)

Modes of Energy (Heat) Transport in Porous Media

- Conduction in solid matrix and in fluid
  - no fluid velocity
  - depends on solid and fluid properties (thermal conductivity)
  - depends in structure of porous matrix

- Convection in fluid
  - may be coupled with conduction in solid matrix and fluid
  - depends on solid/fluid properties and geometry
  - depends strongly on local fluid velocity variations

- Radiation Energy Transport

- heat conduction is transport of energy due to molecular motion of particles (molecules, atoms)
- may take place in solids, liquids and gases
- may be described on a gas kinetic level (microscopic level) as exchange of energy within molecular collisions
- may be described at macroscopic level by Fourier's law

$$\dot{q}_x = k \frac{dT}{dx} \qquad \dot{q} = k \, \nabla T$$

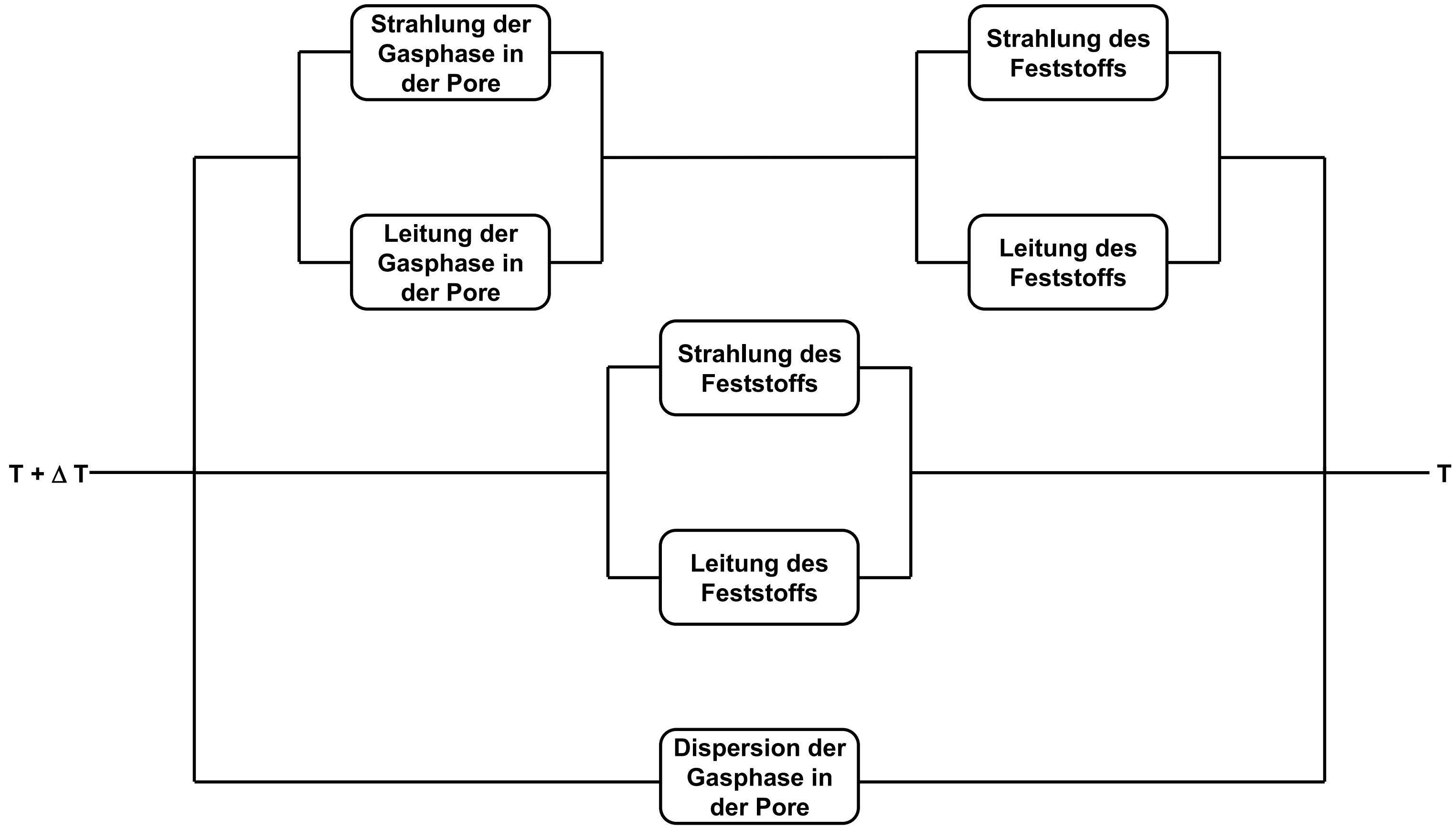
- k thermal conductivity, is a material property
- typical values: air $k_g = 10^{-4}$ $[cal/cm\ sK]$ at $300\ K$
  water $k_f = 10^{-3}$
  steel $k_f = 10^{-1}$

- In porous media, the bulk heat flux is the result of conduction in solid and fluid phase

- For an analysis of heat transport in porous media, effective material properties are used as a result of an a.g. averaging procedure $\langle k \rangle$

  - depends on material properties of each phase
  - depends strongly on solid structure, since $k_{solid} >> k_{gas}$
  - in non-consolidated structures, depends on contact resistance, and thus surface properties (coatings, oxidations, ...)

Heat transport in porous media



Transport equations

$$\frac{\partial(\rho v_j)}{\partial x_j} = 0$$

mass

$$\frac{\partial}{\partial x_j}\left(\rho v_j v_i - \mu \frac{\partial v_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial v_j}{\partial x_i}\right) + \rho g_i$$

momentum

$$\frac{\partial}{\partial x_j}\left(\rho v_j c_P T_g - \lambda_g \frac{\partial T_g}{\partial x_j}\right) = \sum_{k=1}^{N_S} \dot{\omega}_k \Delta h_{j,k}^0 + \frac{\partial}{\partial x_j}\left(\sum_{k=1}^{N_S} \rho h_k D_k \frac{\partial Y_k}{\partial x_j}\right)$$

energy

$$\frac{\partial}{\partial x_j}\left(\rho v_{s,j} Y_k - \rho D_k \frac{\partial Y_k}{\partial x_j}\right) = \dot{\omega}_k$$

species

Modification of transport equations

$$\frac{\partial\left(\rho v_{s,j}\right)}{\partial x_j} = 0$$

Mass balance, s: „superficial velocity“

$$\frac{dp}{dx_i} = -\frac{\mu}{k_{1,ij}} v_{s,j} - \frac{\rho}{k_{2,ij}} \left|v_{s,j}\right| v_{s,j}$$

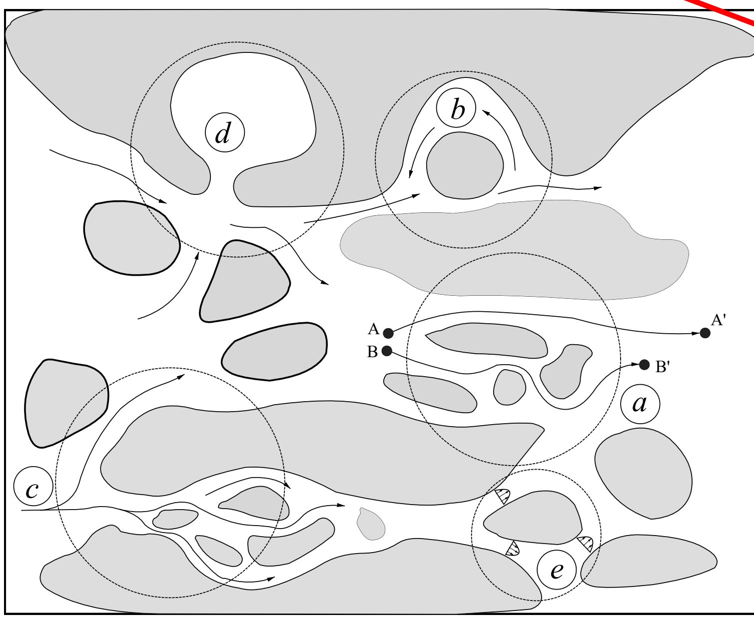
additional pressure loss

$$\frac{\partial}{\partial x_j}\left(\rho v_{s,j} Y_k - \rho \varepsilon D_{k,eff} \frac{\partial Y_k}{\partial x_j}\right) = \dot{\omega}_k$$

conservation of species, influence of dispersion

Einfluss der Dispersion auf den Diffusionskoeffizienten

$$\frac{\partial}{\partial x_j}\left(\rho\varepsilon U_j Y_k - \rho D_{k,eff}\frac{\partial Y_k}{\partial x_j}\right) = \varepsilon\dot{\omega}_k$$



Verschiedene Dispersionsmechanismen in porösen Medien Boukhezar, N.; (2004)

Dispersion wird hervorgerufen durch

a) Tortuosität des porösen Mediums
b) Rezirkulationen
c) unterschiedliche Zugänglichkeit und Durchströmungslängen von Poren
d) nicht durchströmbaren Poren
e) Inhomogenitäten im Strömungsfeld

Heterogeneous model

Gas phase:

$$\frac{\partial}{\partial x_j}\left(\rho v_{s,j} c_P T_g - \lambda \varepsilon \frac{\partial T_g}{\partial x_j}\right) = \sum_{k=1}^{N_S} \dot{\omega}_k \Delta h^0_{j,k} + \frac{\partial}{\partial x_j}\left(\sum_{k=1}^{N_S} \rho h_k \varepsilon D_k \frac{\partial Y_k}{\partial x_j}\right) - \alpha A_V \left(T_g - T_s\right)$$

exchange

Solid phase:

$$\frac{\partial}{\partial x_j}\left(-\lambda_s (1-\varepsilon) \frac{\partial T_s}{\partial x_j}\right) = \alpha A_V \left(T_g - T_s\right) + S_R$$

Source-/Sink term from radiation model

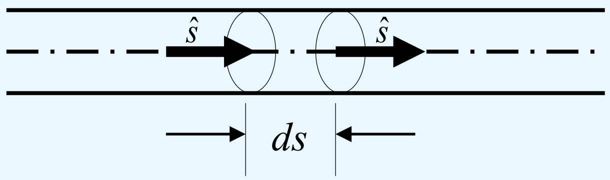
Radiation transfer equation

Radiation transfer equation:

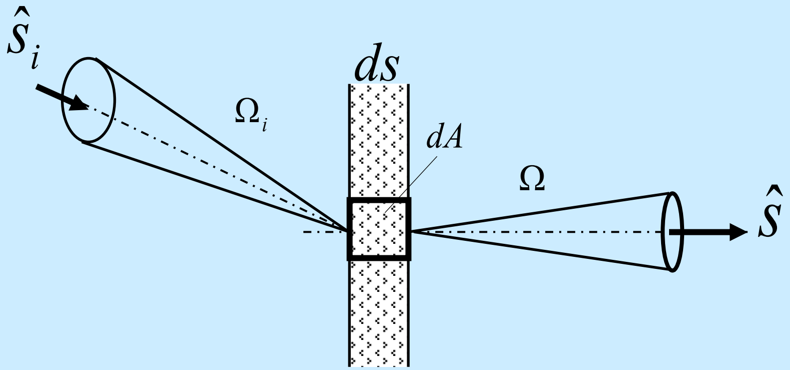
$$I_\eta(s+ds,\hat{s},t+dt)-I_\eta(s,\hat{s},t)=$$

$$\underbrace{\kappa_\eta I_{b\eta}(s,t)ds}_{\textit{Increase due to emission}} - \underbrace{\kappa_\eta I_\eta(s,\hat{s},t)ds}_{\textit{Decrease due to absorption}} - \underbrace{\sigma_{s\eta} I_\eta(s,\hat{s},t)ds}_{\textit{Decrease due to outgoing radiation}} + \underbrace{\frac{\sigma_{s\eta}}{4\pi}\int_{4\pi} I_\eta(\hat{s}_i)\Phi_\eta(\hat{s}_i,\hat{s})d\Omega_i ds}_{\textit{Increase due to incoming Dispersion}}$$

Bilanzierung über Strahlungsrichtungen:



Balance of radiation energy in one direction $\hat{s}$



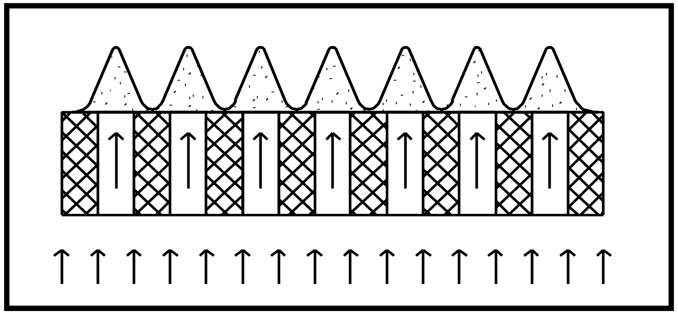
Change of direction due to dispersion

Source term for energy equation: $S_R = \nabla \cdot q_R(\vec{r}) = k(4\pi \cdot I_b - G)$

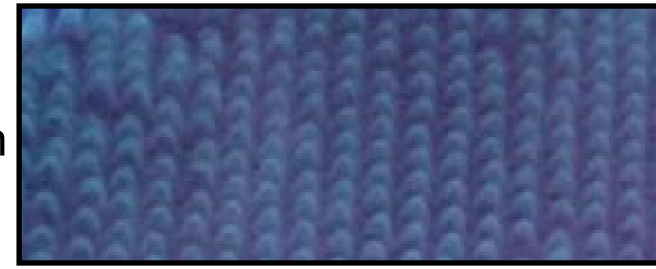
mit $I_b = \frac{\sigma T_b^4}{\pi}$ und $G(\vec{r}) = \int_{4\pi} I(\vec{r},\hat{s})\,d\Omega$

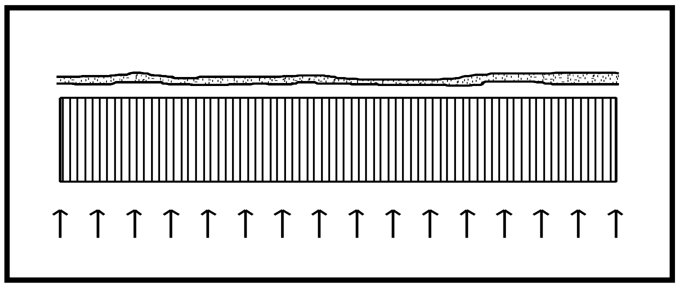
LSTM Erlangen

Stabilisierung des Verbrennungsprozesses an einer porösen Oberfläche und in einem porösen Medium

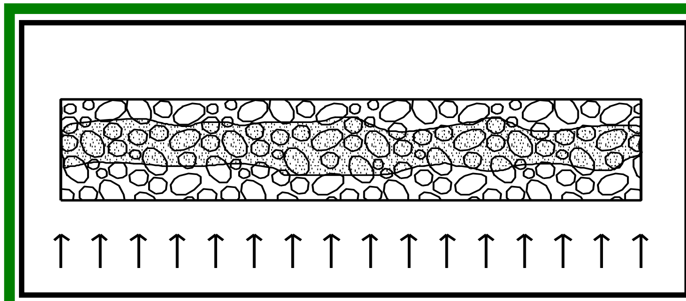


Einzelne Flammen

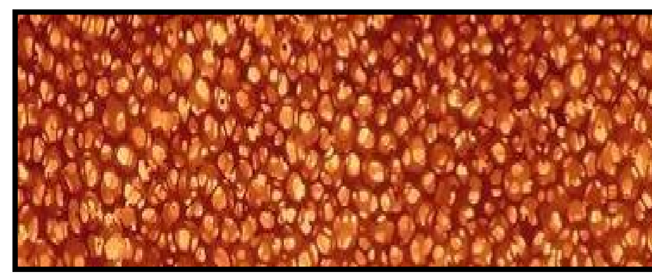




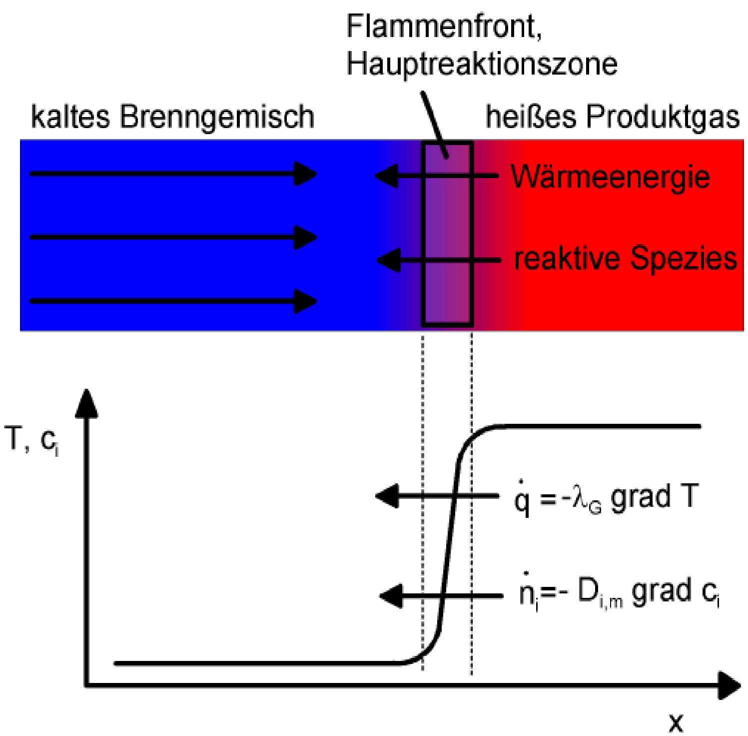
Flammenteppich



Verbrennung im porösen Medium



Mechanismen der Flammenausbreitung



Wärmetransport

Stofftransport

Die Flammenfront breitet sich hauptsächlich durch Wärme- und Stofftransportmechanismen aus

Die **laminare Brenngeschwindigkeit s_L** ist die minimale Anströmungsgeschwindigkeit, damit sich eine laminare Flamme nicht entgegen der Strömung ausbreitet.
Sie wird durch den Wärme- und Stofftransport limitiert.

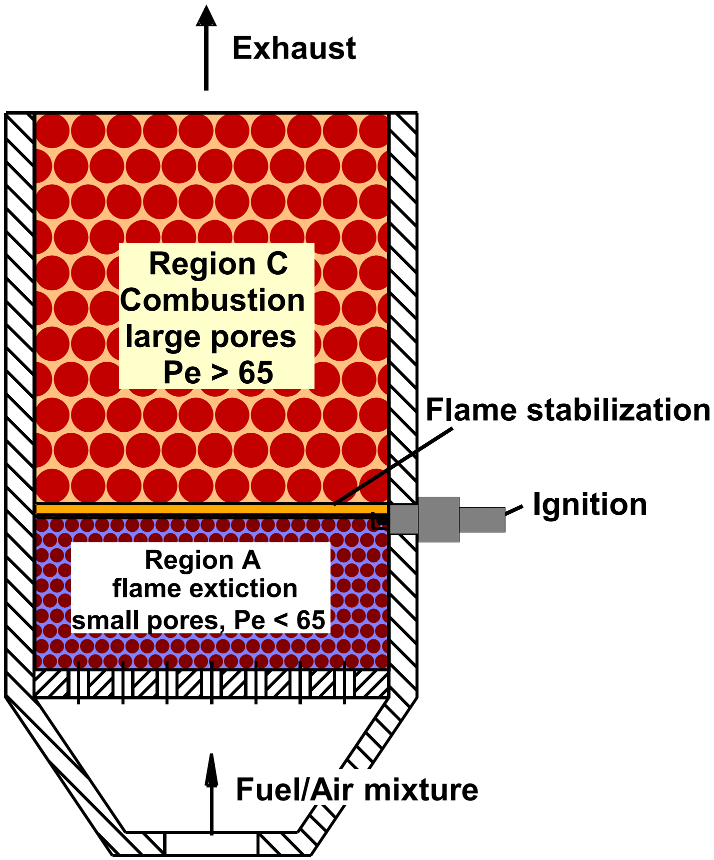
Stabilization of the combustion process in porous media

Flame propagation at a modified Péclet number condition:

$$Pe \geq 65$$

Babkin et al (1991)

$$Pe = \frac{s_L \, d_{p,eff} \, \rho_f \, c_{p,f}}{\lambda_f} = \frac{s_L \, d_{p,eff}}{a_f} = \frac{\text{Heat production}}{\text{Heat removal}}$$



s_L : laminar flame speed

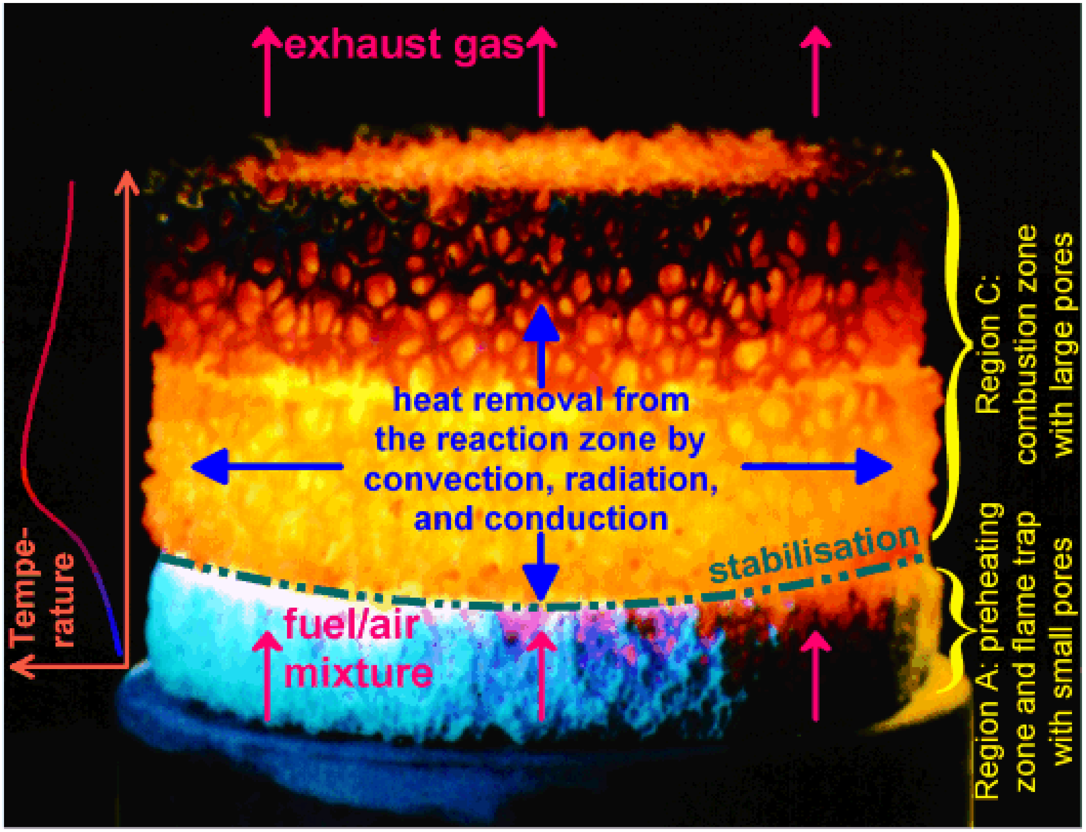
$d_{p,eff}$: equivalent pore diameter

$c_{p,f}$: heat capacity of the gas mixture

ρ_f : density of the gas mixture

λ_f : heat conductivity of the gas mixture

Combustion
in porous inert media



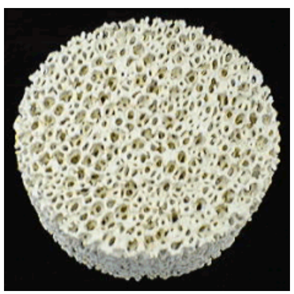
ADVANTAGES:

- adjustable, homogeneous temperature level
- low waste gas emissions
- high combustion stability
- immense power modulation range
- compact size
- complex combustion chambers geometries possible

LSTM Erlangen

Materials for porous burners

Zirconia (ZrO_2)



$T_{max} \approx$ 2.400°C
λ_{solid}: -
T-expans.: +
T-shock res.: -

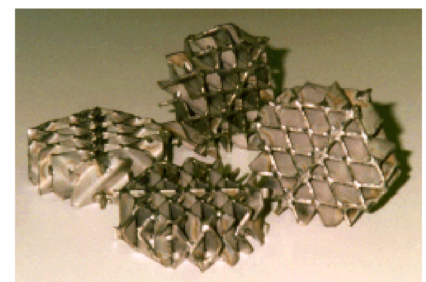
Aluminum oxide (Al_2O_3)

$T_{max} \approx$ 1.950°C
λ_{solid}: +-
T-expans.: +-
T-shock res.: +-

Silicon carbide (SiC)

$T_{max} \approx$ 1.600°C
λ_{solid}: +
T-expans.: --
T-shock res.: ++

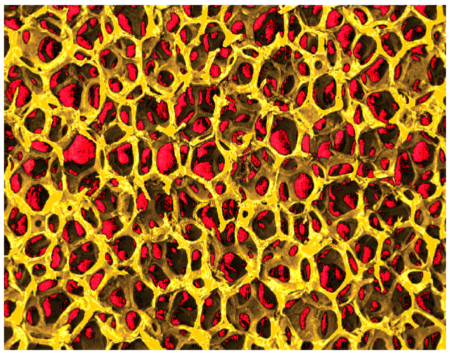
Fe-Cr-Al- and Ni-base alloys



$T_{max} \approx$ 1.250°C
λ_{solid}: +
T-expans.: ++
T-shock res.: ++

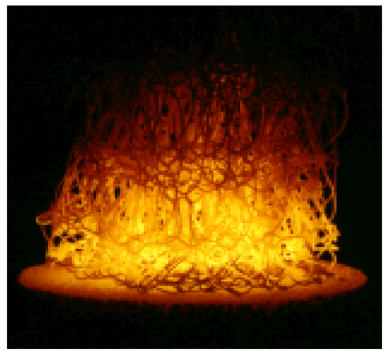
Geometries of porous bodies

Foams



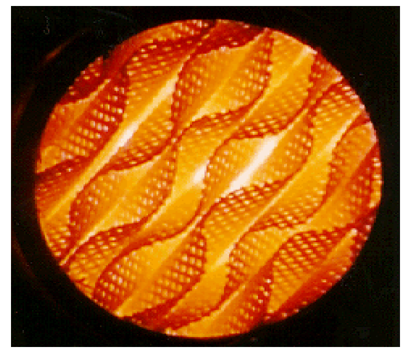
Heat cond.: +
Start up: -
Radiation: +
Dispersion: +-
Δp: -
T_{max}: --
T-shock res.: -

Meshes



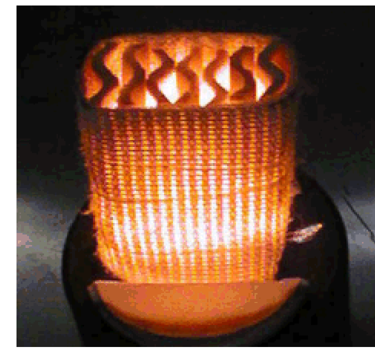
Heat cond.: --
Start up: ++
Radiation: ++
Dispersion: --
Δp: ++
T_{max}: --
T-shock res.: ++

Static mixers



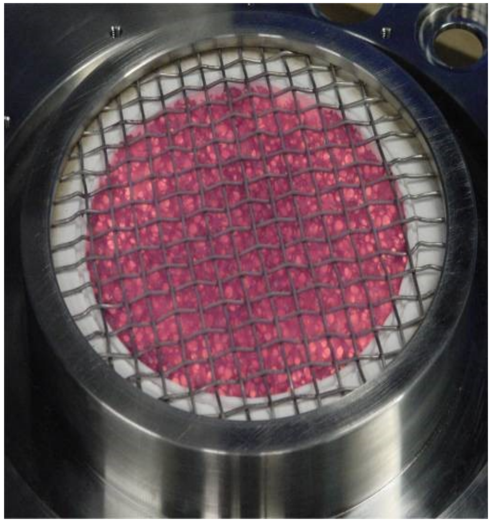
Heat cond.: -
Start up: ++
Radiation: +
Dispersion: ++
Δp: ++
T_{max}: +
T-shock res.: ++

Foam based mixer structures

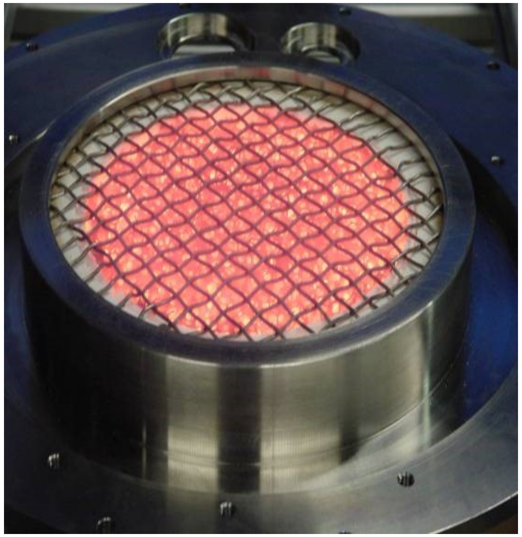


Heat cond.: -
Start up: ++
Radiation: +
Dispersion: ++
Δp: +-
T_{max}: ++
T-shock res.: +

Burner characteristics



2 kW



7 kW

22 kW

Operational parameters:

air ratio λ = 1.2 - 2
power modulation range
P = 2 - 22 kW

Stable combustion
Low emissions